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14. ABSTRACT

By employing the piecewise modal method (PMM) and local equivalent linear stiffness method (LELSM) to estimate NNM frequencies and mode shapes, improved reduced order models were obtained for several types ofo nonsmooth nonlinearities. The resulting approximate frequencies and mode shapes were compared with the exact NNM frequencies and best-fit lines to the NNM manifolds obtained via least squares regression from the direct numerical integration of the full model. The technique was applied specifically to a system with a bilinear clearance-type nonlinearity, to systems with symmetric nonsmooth nonlinearities, and to systems with multiple surfaces of discontinuity. The two methods of approximation (PMM and LELSM) were each shown to work better than linear-based order reduction in all cases. It was found that the PMM method was preferable for bilinear clearance nonlinearities while the LELSM method was preferable for symmetric nonlinearities such as deadzone, saturation, and bang-bang. The PMM and LELSM methods were also used to helop control a piecewise linear system by designing an active controller which forces the system to have certain assigned vibration frequencies and mode shapes.

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ORDER REDUCTION OF LARGE SCALE SYSTEMS VIA NONLINEAR NORMAL MODES

AFOSR GRANT #F49620-01-1-0388

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FINAL REPORT

FINAL REPORT

Project Objectives

a. Obtain and characterize the nonlinear normal modes of general time-varying and nonsmooth systems.

b. Modify existing nonlinear-based order reduction methods to account for nonsmooth nonlinearities and time-varying coefficients. Obtain reduced order models in both state-space and structural form.

c. Investigate alternative order reduction procedures for nonlinear systems such as equivalent linear models, describing functions, and singular perturbation techniques.

d. Compare accuracy of reduced order models with that of reduced models obtained via linear-based techniques.

e. Demonstrate the theoretical methods by applying to some low-order examples involving discontinuous nonlinearities.

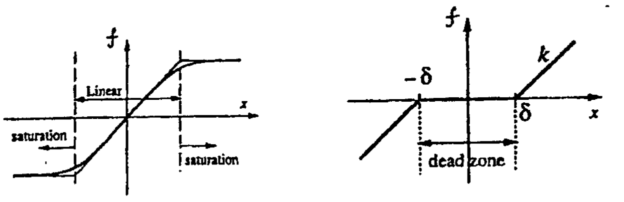
Accomplishments

Many structural systems are modeled using the finite element technique. In this process, the dynamic response problem is reduced to a large set of differential equations. These equations may be linear or nonlinear where nonlinearities can arise from geometry or material behavior. For the purpose of modal analysis and control, only a few dominating modes are important. Therefore, reduced order models that approximate the dynamics of the original large-scale system using linear as well as nonlinear reduction techniques are needed. In order to construct a nonlinear order reduction technique for nonlinear systems, the concept of nonlinear normal modes (NNMs) may be utilized. As suggested by Shaw and Pierre [1], the NNMs are defined as motions occurring on invariant manifolds, which are generally tangent to the corresponding eigenvectors of the linearized system at the equilibrium position and can be obtained analytically in a series form by various techniques [2]. The invariance property ensures that any motion starting exactly in a given modal manifold remains in the same manifold. Model reduction using NNM coordinates is advantageous because one may use fewer NNMs than linear modes to perform equally accurate modal analysis of nonlinear systems. During the course of the project the investigators have focused on two areas of application of order reduction: autonomous systems with nonsmooth nonlinearities (in structural form) and time-periodic nonautonomous systems with smooth nonlinearities (in structural or state-space form).

Existing order reduction methodologies may be classified according to their primary application in either structural dynamics or control theory. The most widely-employed technique designed for structural systems is Guyan reduction [3]. This method, which is presently built into many finite element codes, is a static reduction method since only the system stiffness is taken into account to form the relation between the coordinates to be retained in the reduced model (called the 'master' coordinates) and those to be eliminated (called the 'slave' coordinates). Since the inertia properties are not used in this relation, the slave coordinates are usually chosen to be those with the least amount of mass, measured according to some metric such as the mass condensation

techniqe [4]. Thus the retained master coordinates, which are a subset of the physical coordinates, correspond to the lowest eigenfrequencies of the full model. However, because the reduction is not exact (inertia is neglected), the resulting eigenfrequencies and eigenmodes of the reduced model are only approximations to those in the full model. A variety of techniqes for order reduction in control theory are available, and these are usually applicable to systems in state-space form. In addition, the reduction is usually accomplished in modal form so that the retained coordinates in the reduced model are not a subset of the physical coordinates. Thus these techniques are not widely used in finite element codes.

Nonsmooth nonlinearities such as those depicted in Fig. 1(a-b) exist in many mechanical systems either by design or as the result of wear or failure. Accurate reduced



Saturation

Dead-zone

Figure 1a

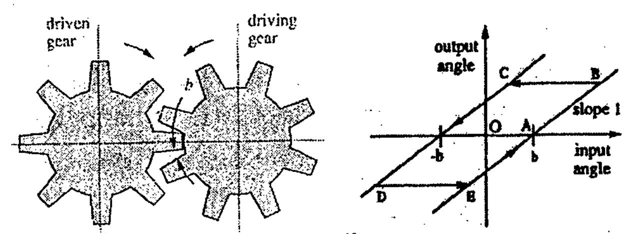


Figure 1b

models of large dimensional nonsmooth systems are an invaluable aid in the analysis and control of such systems. The subject of order reduction of nonsmooth systems has been studied very recently. The recent studies of Rhee and Burton [5] and Jiang et al [4] employed two very different methods. The former study applied a linear transformation, which is an extension of Guyan reduction that accounts for inertia as well as stiffness properties and was obtained by neglecting the nonlinearities in the system. Such a linear-based order reduction transformation, although accurate for linear systems since the exact eigenstructure is preserved in the reduced model and easy to apply to large-scale systems, becomes less accurate as the vibration amplitude increases due to the neglected nonlinearities. In the latter paper a nonsmooth Galerkin-based reduction transformation which takes the nonlinearity into account was applied in order to obtain the invariant manifolds of the corresponding nonlinear normal modes (NNMs) and their associated dynamics for each mode. However, the reduced model is obtained in terms of amplitude-phase coordinates of the modal form instead of the physical coordinates. In addition, large numbers of algebraic equations must be solved (even for low order systems) and the method cannot be easily extended to systems with multiple nonsmooth nonlinearities or several surfaces of discontinuity. These characteristics make the method impractical for use in large-scale structural dynamics and finite element applications.

In a recent paper [7], an alternative technique has been proposed for order reduction of piecewise linear conservative structural systems via nonlinear normal modes approximations. By employing the piecewise modal method (PMM) and local equivalent linear stiffness method (LELSM) to estimate NNM frequencies and mode shapes, improved reduced order models were obtained for several types of nonsmooth

nonlinearities. The resulting approximate frequencies and mode shapes were compared with the exact NNM frequencies and best-fit lines to the NNM manifolds obtained via least-squares regression from the direct numerical integration of the full model. This technique was applied specifically to a system with a bilinear clearance-type nonlinearity ~~in~~ [8], to systems with symmetric nonsmooth nonlinearities ~~in~~ [9], and to systems with multiple surfaces of discontinuity, ~~in~~ [10]. The two methods of approximation (PMM and LELSM) were each shown to work better than linear-based order reduction in all cases. It was found that the PMM method was preferable for bilinear clearance nonlinearities while the LELSM method was preferable for symmetric nonlinearities such as deadzone, saturation, and bang-bang. The success of these these techniques depends on their ability to approximate a nonlinear system with a linear one. Thus, it works best for systems in which the linear part of the model dominates in the dynamics, with the same number of NNMs as degrees of freedom and the invariant manifolds "straight enough" to be approximated by lines (eigenvectors). Hence, the best parameter choices are those that result in weak nonlinearities. However, the range of clearance values in not limited by these methods. The advantages of the technique include a reduced model which uses a subset of the original physical coordinates, contains the form of the nonsmooth nonlinearity of the full model, and can easily accommodate multiple nonsmooth nonlinearities with several surfaces of discontinuity through the use of an equivalent linear stiffness matrix. Figures 2-5 show the frequencies of reduced order models of a two degree-of-freedom system with clearance (Fig. 2), deadzone (Fig. 3), saturation (Fig. 4), and bang-bang (Fig. 5) nonlinearities in a) mode 1 and b) mode 2 computed via linear-based reduction (short-dashed), PMM (solid), LELSM (long-dashed), and numerical simulation of the full model (dots). It is seen that the LELSM method is far more accurate for the symmetric nonlinearities while PMM is more useful only for the clearance mode 1 case.

The PMM and LELSM methods described above were also used to help control a piecewise linear system by designing an active controller which forces the system to have certain assigned vibration frequencies and mode shapes. While the process of assigning frequencies to a controlled system is essentially the familiar technique of pole-placement, assigning both the frequencies and the mode shapes (i.e. eigenstructure) of a system is known as eigenstructure assignment [11], which is usually applied to linear systems by selecting appropriate constant gains for proportional position feedback. If one desires to implement these techniques for systems with nonsmooth piecewise linear nonlinearities, however, the application of constant-gain proportional feedback control is not straightforward because the exact NNM manifolds and frequencies of the uncontrolled system are not known. While it may be possible to implement various non-linear control strategies including gain-switching at the crossing boundaries between the linear subregions, these may be difficult to accomplish in practice because of the inaccuracies caused by control delays or sensor errors or because of the lack of cost-effectiveness of such strategies. In [12-14], three strategies were suggested for eigenstructure assignment of piecewise linear structural systems. The first strategy requires determining n constant actuator gain for an n degree-of-freedom system using PMM, the second involves finding a single degree-of-freedom reduced model with one actuator gain to be controlled, and the third method allows the designer to specify the entire eigenstructure by using a full

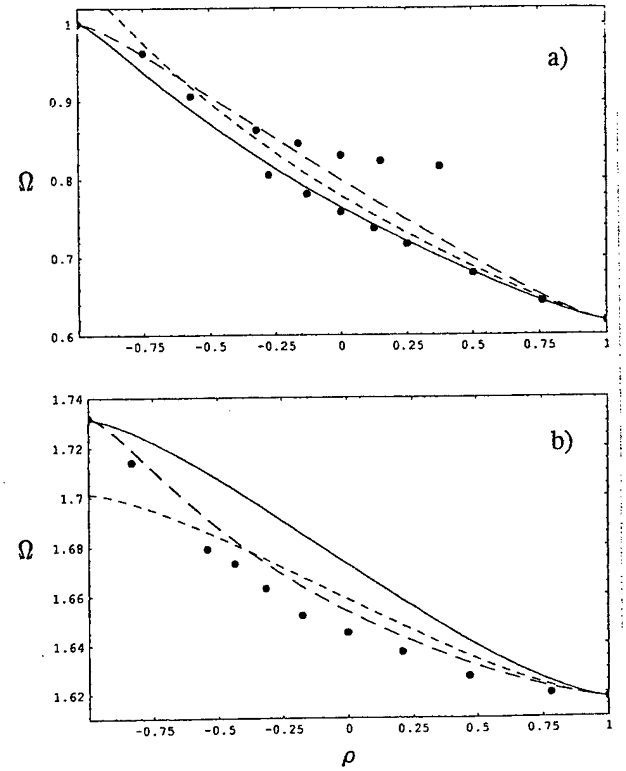


Figure 2

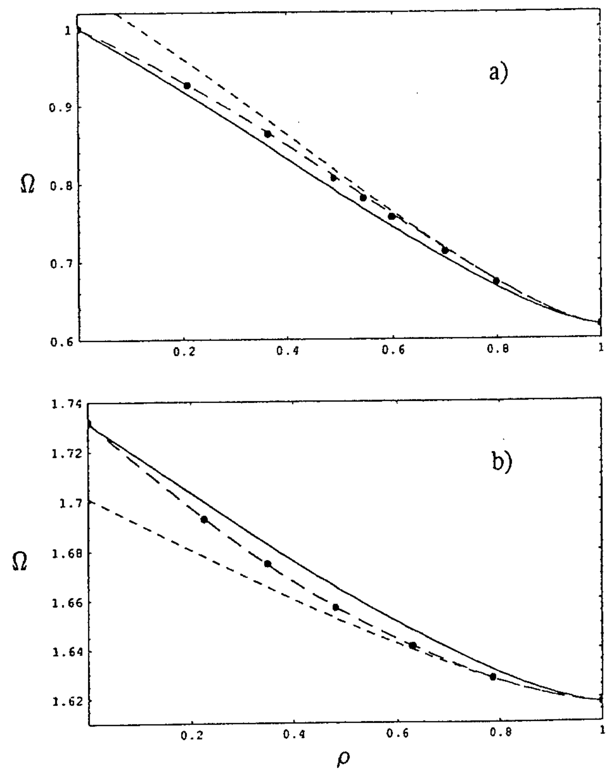


Figure 3

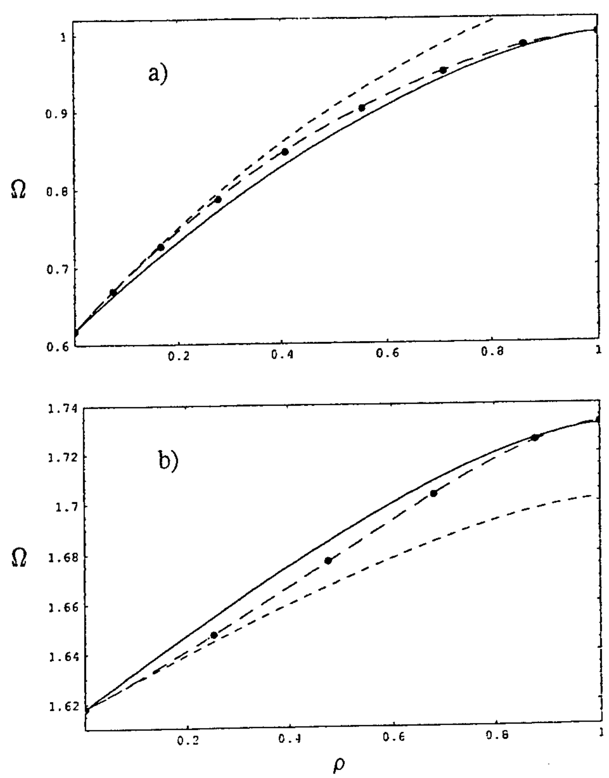


Figure 4

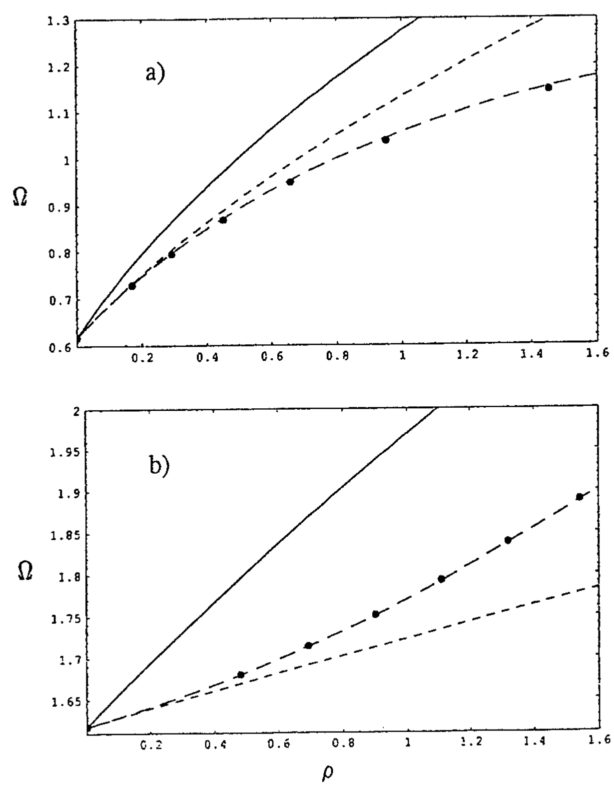


Figure 5

n x n matrix of constant actuator gains. These techniques were applied to systems with a bilinear clearance nonlinearity and a symmetric deadzone nonlinearity and the resulting frequencies and mode shapes were compared by direct numerical simulation. Tables 1-2 compare the theoretical and exact controlled and uncontrolled frequencies and mode shapes for a two degree-of-freedom system with clearance nonlinearity via PMM and order reduction (Table 1) and with deadzone nonlinearity via LELSM (Table 2).

Controller	Iteration	g_i	$\hat{\Omega}_i$	Exact W_i
PMM/ 2 gains	1	0.182, 0.073	0.823, 1.650	
Uncontrolled frequencies: 0.717, 1.618	2	0.243, 0.046	0.843, 1.650	
	3	0.260, 0.039	0.848, 1.650	
Desired frequencies: 0.850, 1.650	4	0.266 0.035	0.850, 1.650	0.865, 1.653
Order Red./ 1 gain	1	0.187	0.827	
Uncontrolled frequencies:	2	0.227	0.847	
0.717, 1.618	3	0.233	0.849	
Desired frequency: 0.850	4	0.235	0.850	0.846, 1.643

Table 1

	Uncontrolled	Desired	Controlled
NNM Frequencies	$\Omega_1 = 0.712$ $\Omega_2 = 1.618$	$\Omega 1 = 0.85$ $\Omega 2 = 1.65$	$\Omega_1 = 0.834$ $\Omega_2 = 1.643$
NNM Mode shapes via regression	$\Phi_1 = \begin{pmatrix} 1 \\ 0.665 \end{pmatrix}$ $\Phi_2 = \begin{pmatrix} 1 \\ -1.618 \end{pmatrix}$	$\Phi_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\Phi_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\Phi_1 = \begin{pmatrix} 1 \\ 0.988 \end{pmatrix}$ $\Phi_2 = \begin{pmatrix} 1 \\ -1.004 \end{pmatrix}$

Table 2

An important class of problems, such as those depicted in Figure 6, gives rise to nonlinear differential equations with time-periodic coefficients. Recently, order

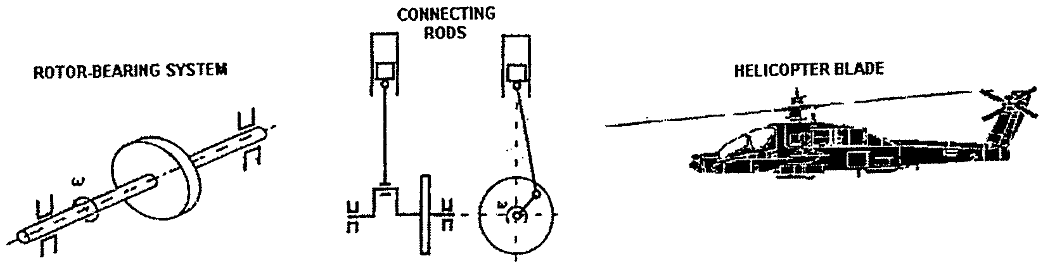
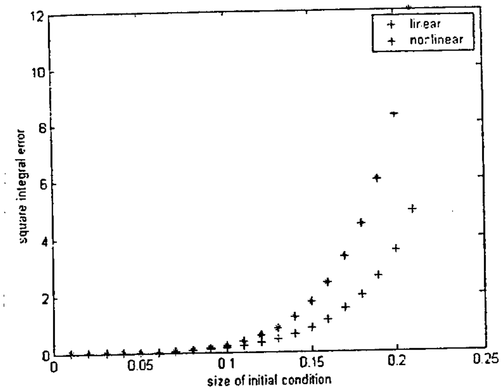


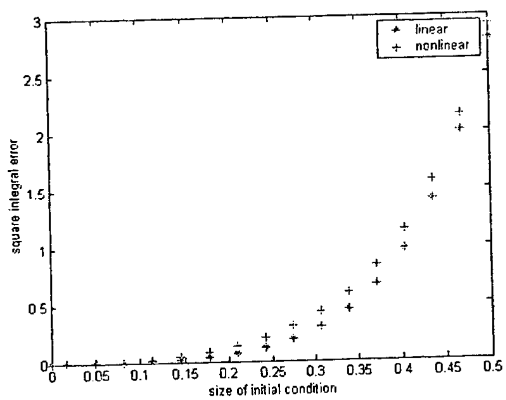
Figure 6

reduction techniques for linear time-periodic systems have been reported in [9-11]. While the first study applied Krylov techniques and time-varying Pade approximation from a control point of view, the second study simply neglected the contribution of the periodic terms. In general, such reduced models cannot portray the correct dynamics of the original periodic systems. For a more meaningful approach, the contribution of the periodic terms were included in [11] through the use of the Liapunov-Floquet (L-F) transformation that converts a time-varying linear system matrix into an equivalent time-invariant one. However, a similar approach to nonlinear time-periodic systems may not yield acceptable results. Recently, it has been suggested in [12-15] that a nonlinear order reduction is possible for time-periodic systems through an application of the L-F transformation and an extension of the invariant manifold concept. As in the case of "Time-Periodic Center Manifold Theory", where stable states are expressed as periodically-modulated nonlinear functions of critical states, it is proposed in these studies to express the slave (non-dominant) states as periodically-modulated nonlinear functions of the master (dominant) states. This generalization of the idea suggested by Shaw and Pierre [1] for autonomous systems implies the existence of invariant manifolds for the time-periodic nonlinear system. In addition, the reducibility condition associated with the master-slave relation provides more general definitions and interpretations of various types ('parametric', 'conventional internal', 'true internal', 'conventional combination', 'true combination') of resonances encountered in parametrically excited systems. Unlike perturbation or averaging approaches, the parametric excitation term is not assumed small. Furthermore, the method can be implemented in both the state-space and structural (second order) forms. Recent results in the form of time traces, Poincare maps, and plots of the integral-square-error for two parametrically excited coupled pendulums indicate that the proposed order reduction technique is more accurate than linear-based order reduction, particularly for higher vibration amplitudes. Figures 7-9 show the integral square error of the nonlinear and linear-based reduced order models compared with the exact solution of the full model for various sizes of the initial condition for the cases of no resonance (Fig. 7), true internal resonance (Fig. 8), and true combination resonance (Fig. 9). It is seen that in the absence of resonance, the nonlinear-based reduced model is more accurate, while the linear-based one is more accurate in the presence of resonances. This implies that multi-mode nonlinear-based reduced models must be obtained for larger order systems under such resonance conditions to preserve accuracy.



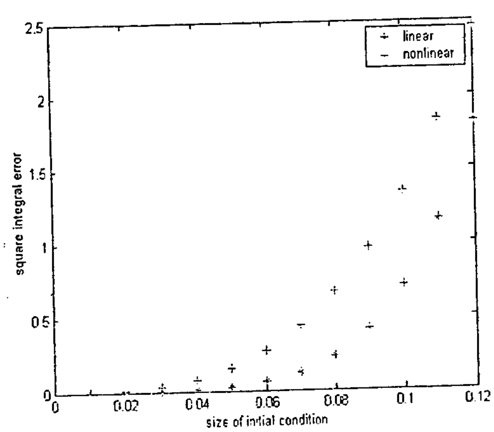
Integral Square Error for gradually increasing initial conditions for Double Inverted Pendulum away from resonance.

Figure 7



Integral Square Error for gradually increasing initial conditions for Double Inverted Pendulum in 1:1 internal resonance

Figure 8



Integral Square Error for gradually increasing initial conditions for Double Inverted Pendulum in combination resonance

Figure 9

Two more efficient alternatives to the method described above based on obtaining an approximation to the time-periodic invariant manifold are based on using singular perturbation (SP) and post-processing. In the SP method, the nonlinear differential equations of motion are rewritten as a classical singular perturbation problem by partitioning the system dynamics into slow and fast states and ordering it according to the decay rates. In the limit as the rates of the slave states goes to zero, a differential-algebraic system (DAE) is obtained which is a reduced order approximation of the original large-scale system and can be solved by standard DAE techniques such as fixed-point iteration. In post-processing, a correcting factor which "uplifts" the "flat" manifold is employed using iterative schemes. The main advantage here is the computational effort is much less than in computing the invariant manifold and somewhat less than in SP. However, as is to be expected, the error due to each of the three methods varies inversely with computational effort, so that the invariant manifold approach is most accurate, followed by the SP method and finally the post-processing approach as shown in Figure 10. Finally, it has also recently been demonstrated how this technique can be extended to the case of periodic/quasiperiodic systems in which the linear part is periodic with some principal period and the nonlinear part is periodic or quasiperiodic with frequencies which are incommensurate with the principal frequency of the linear part. Thus, while the L-F transformation may be employed to transform the linear part as usual, the subsequent nonlinear analysis results in complicated resonance conditions involving more than one parametric frequency [16].

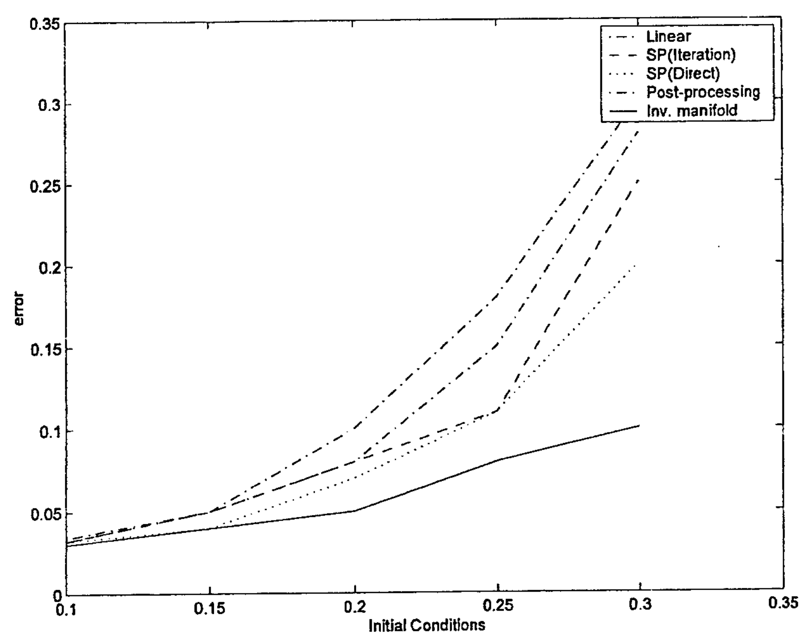


Figure 10

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Publications and Conference Proceedings Resulting From the Project

Sinha, S. C., S. Redkar, V. Deshmukh, and E. A. Butcher, "Order Reduction of Parametrically Excited Systems: Techniques and Applications," *Nonlinear Dynamics,* in press.

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Sinha, S. C., S. Redkar, and E. A. Butcher, "Order Reduction of Nonlinear Systems with Time-Periodic Coefficients Using Invariant Manifolds," *J. Sound and Vibration,* in press.

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